Published by Institute of Physics Publishing for SISSA/ISAS

Received: *April 4, 2002*
Accepted: *December 19, 2002*

Comments on cosmological RG flows

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Abstract: We study cosmological backgrounds from the point of view of the dS/CFT correspondence and its renormalization group flow extension. We focus on the case where gravity is coupled to a single scalar with a potential. Depending on the latter, the scalar can drive both inflation and the accelerated expansion (dS) phase in the far future. We also comment on quintessence scenarios, and flows familiar from the AdS/CFT correspondence. We finally make a tentative embedding of this discussion in string theory where the scalar is the dilaton and the potential is generated at the perturbative level.

Keywords: Bosonic Strings, Superstrings and Heterotic Strings, AdS-CFT and dS-CFT Correspondence, Cosmology of Theories beyond the SM.

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1. Introduction

Recent experimental data can be interpreted in terms of the presence in our universe of a positive cosmological constant (see [1] for a review). Its magnitude is however very small with respect to natural quantum gravity units, $\Lambda \sim 10^{-120} m_p^4$. Nevertheless, a positive constant energy density is bound to dominate the evolution of the universe in the far future. The future evolution will thus be very close to a de Sitter universe. Note that since only the expanding de Sitter universe is relevant to these considerations, we can assume that it has open spatial sections. We can thus write the dS metric in the following most useful form:

$$ds^2 = -dt^2 + e^{2Ht} dx_i^2 \,. \tag{1.1}$$

Here t is the cosmological comoving time and H is the Hubble constant (which, for de Sitter, is *really* constant) associated to Λ as $H^2 \sim \Lambda/m_p^2$. A universe with a positive cosmological constant will thus have a metric asymptotic to (1.1) for $t \to \infty$.

An accelerated (de Sitter like) expansion is also present in the inflationary scenario. This expansion stage is supposed to have taken place in the very early universe and is useful to explain various features of the present universe (most notably its flatness). We can thus suppose that during this stage the universe also had a metric similar to (1.1), however with a different Hubble constant H. For our following considerations, let us just assume that in contrast with the "future" H, the "inflationary" H is (within a few orders of magnitude) of the Planck scale. Note here that the inflationary de Sitter phase in the past has not to be confused with a de Sitter universe near its past infinity: such a contracting universe is the time reversal of (1.1) and is not relevant to (traditional) cosmological considerations. So to cut the story short, the universe has expanding de Sitter phases both in the past and the future, however there is an enormous hierarchy between the two cosmological constants driving the expansion.

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Given these observations based on experimental data, the question arises of how to incorporate them into string theory. Setting aside conceptual problems related to quantum gravity in de Sitter space [2, 3], an interesting point of view is drawn from the analogy with the AdS/CFT correspondence [4], where it is postulated that a gravitational theory in the bulk of de Sitter spacetime is dual to a euclidean CFT situated on its future boundary [5] (we consider here the set up of the correspondence where the metric is (1.1), which covers half of de Sitter spacetime and reaches only to one boundary). A step further in the analogy is to associate the cosmological time coordinate t with renormalization group flow in the dual CFT [6]: time evolution in de Sitter corresponds to scale transformations in the dual CFT. It is then natural to generalize that beyond pure de Sitter and consider renormalization group flows in the boundary theory as cosmological evolution in the bulk. In particular, evolution between two de Sitter phases is interpreted as the RG flow of the dual theory between two fixed points [6], with the UV one (corresponding to the future) having a much larger central charge than the IR one.

Once this correspondence between cosmological evolution in the bulk and RG flow in the boundary field theory is stated, it is interesting to investigate it in many respects. The many AdS/CFT techniques and concepts developed in the context of the domain wall/RG flow correspondence can be translated into dS language, and differences analyzed. Some of the flows considered in AdS space, when mapped to dS, can be actually given a cosmological interpretation. Conversely, cosmological models can be analyzed with field theory insight. The major drawback of dS considerations versus AdS ones, is that in the latter extended supersymmetry allows us to consider very specific potentials which entail precise predictions on the field theory side. In the dS case, for which supersymmetry is necessarily absent (presumably broken), we are left with generic considerations and toy potentials.

In the following, we will firstly formulate the problem of cosmological evolution coupled to a scalar with a potential, and cast the equations of motion in a first order form. This implies that the potential derives from a prepotential. We then analyze generically the field theory interpretation of the extrema of the potential and prepotential, together with some considerations about the scalar field solutions around the extrema and their interpretation in the dual field theory. We then consider a few specific flows, both exact ones and others from the AdS/CFT literature, and comment on their cosmological implications. A particular class of flows is the one generated by potentials which are a sum of exponentials, since they could come from loop corrections to the string effective action. Their cosmological and RG flow interpretation is given. We conclude with a brief discussion.

2. Cosmology with a scalar potential

We are going to consider the simplest model of cosmological evolution, gravity coupled to a scalar with potential. Although a set up with several scalars is more general, the exact analytic flows we want to consider are more likely to arise when only one scalar is active, as for example in the AdS flows presented in [7, 8]. Moreover we will apply in section 5 this formalism to the case where the scalar is the dilaton, while the other moduli are (hopefully) frozen.

We take the spacetime to be $D+1$ dimensional, and we single out the time coordinate t. The action is:

$$S = m_p^{D-1} \int dt d^D x \sqrt{-g} \left(R - \frac{1}{2} \partial \phi^2 - V(\phi) \right) . \tag{2.1}$$

The metric is taken to be isotropic, homogeneous and flat:

$$ds^2 = -e^{2b(t)} dt^2 + e^{2a(t)} dx_i dx_i \, , \qquad i = 1 \ldots D \, . \tag{2.2}$$

We include for the moment a lapse function, to be fixed later to 1. The Einstein equations are thus:

$$\frac{D(D-1)}{2} \dot{a}^2 = \frac{1}{4} \dot{\phi}^2 + \frac{1}{2} e^{2b} V \equiv \frac{1}{2} e^{2b} \rho \, , \tag{2.3}$$

$$-(D-1) \left(\ddot{a} - \dot{a}\dot{b} + \frac{D}{2} \dot{a}^2 \right) = \frac{1}{4} \dot{\phi}^2 - \frac{1}{2} e^{2b} V \equiv \frac{1}{2} e^{2b} p \, , \tag{2.4}$$

where the energy density is defined by $\rho = -T^t{}_t$ and the pressure by $p = T^1{}_1 = \ldots = T^D{}_D$. Note that the positivity of the kinetic term assures the null energy condition $p + \rho \geq 0$, however the dominant energy condition $p \leq \rho$ is satisfied if and only if the potential is non negative $V \geq 0$. This is what we will assume here, indeed it will turn out to be a crucial condition for most of the flows. Note also that it is the opposite condition that is satisfied for AdS flows.

The equation of motion of the scalar field is:

$$\ddot{\phi} + D\dot{a}\dot{\phi} - \dot{b}\dot{\phi} + e^{2b} \partial_\phi V = 0 \, . \tag{2.5}$$

This equation together with (2.3) imply (2.4), so the latter is redundant.

The action can be rewritten using the reduced variables $a(t)$, $b(t)$ and $\phi(t)$:

$$S \propto \int dt e^{Da-b} \left\{ -D(D-1)\dot{a}^2 + \frac{1}{2} \dot{\phi}^2 - e^{2b} V \right\} . \tag{2.6}$$

The energy functional vanishes on-shell due the G_{tt} equation, (2.3).

We now wish to rewrite the equations of motion in terms of first order equations, provided the potential V is of some specified form. That is automatically provided in a supersymmetric theory, here however we have to make a specific ansatz. Mimicking the technique of [9, 10], we suppose the potential is the difference of two squared terms. It turns out that the correct expression in this case is (see also [11] for a recent similar discussion in de Sitter):

$$V = \frac{D}{D-1} W^2 - 2(\partial_\phi W)^2 \, . \tag{2.7}$$

The equations (2.3) and (2.5) then are satisfied when:

$$\frac{1}{2} \dot{\phi} = -e^b \partial_\phi W, \tag{2.8}$$

$$(D-1)\dot{a} = e^b W \, . \tag{2.9}$$

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We have fixed here a sign ambiguity for W. In the following we will mainly use the $b = 0$ gauge. Let us stress that (2.7) is motivated only for finding first order equations from which the equations of motion derive, and not by positive energy considerations. The energy is indeed always vanishing on shell.

It is interesting to note that the expression for the potential (2.7) is "upside down" with respect to its canonical form in supersymmetric (or, equivalently, AdS) theories. Most notably, solutions for which ϕ is constant will have a positive energy density since in that case $V \propto W^2$. This is of course consistent with the fact that we are interested in spacetimes which asymptote to de Sitter.

Let us comment briefly on the c-function and the c-theorem. In analogy with [12, 13], a c-function has been proposed in [6] for the flat homogeneous isotropic case which is $c \sim H^{-(D-1)}$, where $H \equiv \dot{a}$ in our notations (see also [11] for a recent refinement of the proposal). Note that a small cosmological constant, and hence a small Hubble constant H, leads to a large central charge. Conversely, a cosmological constant of the Planck scale leads to a central charge of order unity. The c-theorem follows directly from the Friedmann equations for such a cosmology, $\dot{H} \sim -(\rho + p) \leq 0$ in all generality, thus implying $\dot{c} \geq 0$. This identifies the UV with the future (which is near the dS boundary) and the IR with the past (the deep dS "interior").

Note that if we enforce $V \geq 0$ and thus $p \leq \rho$, we also have an upper bound on the variation of c. This is however somewhat loose. Note first that:

$$\dot{c} = \frac{1}{2}\frac{c}{H}(\rho + p) \leq \frac{c}{H}\rho = D(D-1)cH\,, \tag{2.10}$$

where we have used (2.3). If c saturates the bound on its variation, and we write the "volume" of a spacelike section as $v = e^{Da}$, we get $c \sim v^{D-1}$. Thus c is allowed to grow even faster than the volume of the spacelike section of the universe (for $D > 2$; for $D = 2$, that is 3 dimensional cosmology, c can at most grow as the volume). In some sense, the growth of degrees of freedom in the boundary theory can be bigger than the expansion of the volume in Planck units of a spatial section of the universe.

3. General considerations on the potential

In this section we make general considerations on the extrema of the potential, in relation with the flows and cosmologies to which they correspond.

As it is clear from (2.9), the Hubble constant is proportional to the prepotential. If we want our flow (or our cosmology) to go between two fixed values of ϕ, say $\phi_\pm \equiv \phi(t \to \pm\infty)$, we then immediately see that $W(\phi_\pm) \equiv W_\pm$ must be extrema, and such that $W_- > W_+ > 0$ to comply with $\dot{H} \leq 0$ and $H > 0$ for an expanding universe. Clearly, the generic situation is when ϕ starts rolling down from a maximum in W_- and lands to a minimum in W_+. Of course less generic situations can be thought of, where higher derivatives of W vanish in ϕ_- and/or ϕ_+.

Once stated the above, we have to make sure that the evolution still seems reasonable in terms of the potential V, for instance we want that the universe settles into a minimum of V in order to avoid excessive fine tuning. To do this, we first note that V has two kinds of extrema, those for which $W' = 0$, that is the extrema of the prepotential, and those for which $\frac{D}{D-1}W = 2W''$ (in the following we denote by primes derivatives with respect to ϕ). Following the argument of the previous paragraph, our first order equations are best suited to handle extrema of the first kind. We would thus like to enforce as a first condition that there is no extremum of the second kind in between the two extrema we are considering.

Secondly, let us consider the second derivative of V evaluated at an extremum of W:

$$V''|_{W'=0} = 2W''\left(\frac{D}{D-1}W - 2W''\right). \tag{3.1}$$

Assuming the flow begins at a (positive) maximum of W, then ϕ_- is clearly also a maximum of V. The minimum of W is also a minimum of V provided $\frac{D}{D-1}W_+ > 2W''_+$. This is obviously the same as requiring that there is no extremum of the second kind between ϕ_- and ϕ_+.

Thus the condition for the flow to be acceptable in our framework is that $\frac{D}{D-1}W > 2W''$ throughout the flow[1] (another condition is of course that the potential V is non negative along the flow).

Going now back to the RG interpretation we see that the cosmologically trivial remark that a scalar field has to roll down the potential and stabilize eventually at a minimum, translates into the c-theorem where one starts from a fixed point in the IR (the maximum of both V and W) and then integrates in degrees of freedom at higher and higher energies to eventually reach a fixed point in the UV (the minimum of V and W). We have here of course a flow between two fixed points because we suppose that both extrema are positive.

The c-theorem in dS language can be written as $H_{IR} > H_{UV}$, or $\Lambda_{IR} > \Lambda_{UV}$. This is the same as for AdS flows, if one replaces the cosmological constants with their absolute values. In AdS however, the potential V is really upside down, and thus non positive, so that the IR extremum is lower than the UV one.

We can also analyze the behaviour of the scalar field near the UV extremum. We note immediately that the first order equation (2.8) actually picks up one of the two asymptotic solutions of the scalar equation (2.5).

Generally, if $V''_+ = m^2$ and $V_+ = D(D-1)H$, the scalar equation (2.5) becomes $\ddot{\phi} + DH\dot{\phi} + m^2\phi = 0$ and it has asymptotic solutions:

$$\phi \sim e^{\lambda_\pm t}, \qquad \lambda_\pm = -\frac{DH}{2} \pm \sqrt{\left(\frac{DH}{2}\right)^2 - m^2}. \tag{3.2}$$

It is interesting to note that since ϕ_+ is also an extremum of the prepotential, (3.1) implies that:

$$m^2 = 2W''_+\left(DH - 2W''_+\right) \leq \left(\frac{DH}{2}\right)^2, \tag{3.3}$$

and thus the two roots $\lambda_\pm$ are always real:

$$\lambda_\pm = -\frac{DH}{2} \pm \left|\frac{DH}{2} - 2W''_+\right|. \tag{3.4}$$

[1]Given a potential V one can always try to solve (2.7) for W, with boundary conditions such that the flow is between two $W' = 0$ extrema (see e.g. [14] for a discussion of this issue in AdS/CFT). However the generic solution is not analytic and thus beyond the scope of these considerations.

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Note we always have $\lambda_+ > -\frac{DH}{2} > \lambda_-$, and we can accordingly associate dimensions in the dual CFT as $\lambda_- = -\Delta H$ and $\lambda_+ = -H(D-\Delta)$ (restricting here to the case $\Delta \geq D/2$). The situation where the roots are real [2] is surprising against flat space intuition, but saves us from having to consider complex dimensions and thus an explicitly non unitary boundary CFT [5].

The conventional wisdom in AdS/CFT is that if the solution is normalizable in AdS it corresponds to a flow generated by giving a VEV to an operator of dimension Δ, while if the solution is non normalizable in AdS, the flow corresponds to deforming the theory by the same operator. In dS the situation is more tricky, since normalizability is defined on a spacelike section. However in order to draw an analogy with the AdS case, it seems natural to keep distinguishing solutions according to their normalizability along the time direction (this is equivalent to considering the finiteness of their asymptotic energy, see [15] for a recent related discussion in the IR). We thus wish to associate solutions with $\phi \sim e^{\lambda_+ t}$ with a deformation of the boundary theory with an operator of dimension Δ, while we associate the solutions $\phi \sim e^{\lambda_- t}$ with giving a VEV to the same operator.

It is then straightforward to continue the analogy by mapping minima with $m^2 > 0$ to relevant operators ($\Delta < D$), extrema with $m^2 = V''_+ = 0$ to marginal ones, and maxima with $m^2 < 0$ (actually tachyons) to irrelevant operators. Thus the situation considered before where we evolve to a minimum in the UV is obviously associated with relevant operators in the UV theory.

The first order equation (2.8) picks up the solution:

$$\phi \sim e^{-2W''_+ t}\,. \tag{3.5}$$

It will be non normalizable in the sense discussed above if $W''_+ < \frac{DH}{4}$. In this case it corresponds to a (relevant) deformation of the dual CFT. If on the other hand $\frac{DH}{4} < W''_+ < \frac{DH}{2}$, it will be related to the presence of a VEV.

The marginal case occurs when $V''_+ = 0$. This can happen for 2 reasons, either $W''_+ = 0$ or $2W''_+ = DH$. In the first case, the solution (3.5) is actually power behaved, and corresponds to the deformation by a (presumably not exactly) marginal operator, while the second case corresponds to the VEV of the marginal operator.

Let us make a last general comment. It is interesting to consider the evolution of the equation of state $p = w\rho$ of the scalar field. Near the extrema of the flow, the stress energy tensor is dominated by the potential, and thus the equation of state approaches $p = -\rho$ as for a cosmological constant. One can ask which is the maximum value of w along the flow, knowing that it is bounded by 1 since the potential is non negative. Inserting (2.7) and (2.8) into the definitions of p and ρ, one can write:

$$w = \frac{p}{\rho} = -1 + 4\frac{D-1}{D}\frac{W'^2}{W^2}\,. \tag{3.6}$$

While one can certainly build ad hoc (pre)potentials such that w rises close to 1 at a certain point in the flow, it is striking that limiting oneself to a simple minimum of W

[2] Actually, having real ϕ solutions is equivalent to be able to reformulate the problem with first order equations and a real prepotential.

one cannot get over $w = 0$. Indeed, if ϕ is sufficiently close to the minimum, we can write $W = W_+ + \frac{1}{2}W''_+\phi^2$. The maximum of w, which is attained when $WW'' = W'^2$, implies that $\frac{1}{2}W''_+\phi^2_{\rm max} = W_+$. This leads to $\frac{W'^2}{W^2}|_{\rm max} = \frac{W''}{W}|_{\rm max} = \frac{W''_+}{2W_+} < \frac{D}{4(D-1)}$ (by requiring ϕ_+ to be a minimum of V), which in turn gives $w < 0$. This is actually an interesting feature, since while in this model the scalar field is supposed to drive both inflation and the future accelerating phase, it would be unrealistic if it was to simulate some kind of matter/radiation at intermediate stages. On the other hand, it can be shown that if W has a zero of higher (even) order, the parameters can be tuned so that w rises above 0 at some point of the flow near the minimum. These are flows associated with deformations of the boundary theory by marginal operators.

4. Some cosmological RG flows

Due to the absence of supersymmetry, and thus of a theory providing us with a class of (pre)potentials, we are actually left free to analyze general forms of potentials and the cosmological flows that they give rise. One could actually even take the opposite approach, to choose an interesting (analytic) flow and then derive the potential that generates it.

For instance, a most obvious cosmological evolution interpolating between Hubble constants H_- in the far past and H_+ in the far future is the following:

$$\dot{a} = \frac{H_- + H_+}{2} - \frac{H_- - H_+}{2} \tanh\frac{t}{T}\,, \tag{4.1}$$

where of course $H_- > H_+$, and T is the time scale of the evolution. From (2.9) we thus have an expression of $W(t) \equiv W(\phi(t))$. In combination with (2.8) we can straightforwardly find an equation for the temporal evolution of ϕ, $\dot{\phi} = \sqrt{2|\dot{W}|}$, which is solved by:

$$\cos\xi\phi = -\tanh\frac{t}{T}\,, \qquad \xi^{-2} = T(D-1)(H_- - H_+)\,. \tag{4.2}$$

The scalar field evolves from $\phi_- = 0$ to $\phi_+ = \pi/\xi$. Plugging (4.2) into (4.1), we finally get the prepotential:

$$W(\phi) = (D-1)\left(\frac{H_- + H_+}{2} + \frac{H_- - H_+}{2}\cos\xi\phi\right), \tag{4.3}$$

which has a maximum in ϕ_- and a minimum in ϕ_+. Now in order to satisfy the condition which ensures that we land in a minimum of the potential V as discussed after (3.1), we need to take $T > (DH_+)^{-1}$. This condition states that the age of the universe (more precisely, the characteristic time of the evolution from one de Sitter phase to another) in such a flow is bigger than the scale fixed by the late time cosmological constant $\Lambda_+ \sim H_+^2$. That would give a time scale of (roughly) an order of magnitude or so larger than the assumed age of the universe. Note however that, in agreement with the general argument after (3.6), we find that $w < 0$ along all the flow (and w comes very close to 0 at a point in the flow if $H_+ \ll H_-$). Thus if we suppose that some form of matter and/or radiation dominates the evolution at some intermediate stage, without disrupting the general behaviour of the flow, the timescale T could well be suitably reduced to an acceptable magnitude.

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In the M-theory context a prepotential like (4.3) has been considered for instance in [16], and it can possibly arise in non supersymmetric set ups. However for a prepotential like (4.3) to be cosmologically interesting, one needs to have $H_+ \ll H_-$ and thus to fine tune its minimum to be almost on the ϕ axis.

A very similar story applies to cubic prepotentials, where again one has to fine tune the minimum to be close to the axis. The similarity derives from the fact that for both potentials the two extrema are simple zeroes of W'. Also in this case it is straightforward to find an analytic flow. Thus the simple flow (4.1) seems to be actually a good representative of a rather general class of flows.

A different class of prepotentials leading to an analytic flow derives from the analogy with the flow analyzed in [7]. In its original AdS version, this flow represents a mass deformation of $N = 4$ SYM away from its UV fixed point towards pure $N = 1$ SYM in the IR. The confining behaviour of the latter is translated into the singularity at finite radius of the (analytic) domain wall solution.

Reverting to our de Sitter framework, we can write this class of potentials as:

$$W = g + h\ \cosh \alpha\phi\,, \tag{4.4}$$

where $h > 0$ and $g + h > 0$. The condition for the potential V to be positive everywhere (and thus to have only one minimum at $\phi = 0$) is that $\frac{D-1}{D}2\alpha^2$ must be smaller than 1 if $g \geq 0$ or smaller than $1 + \frac{g}{h}$ if $g < 0$.

Choosing the gauge $b = 0$, the equations (2.8)-(2.9) become:

$$\frac{1}{2}\dot{\phi} = -\alpha h \sinh \alpha\phi\,, \tag{4.5}$$

$$(D-1)\dot{a} = g + h\ \cosh\alpha\phi\,. \tag{4.6}$$

The solution to (4.5) is:

$$e^{-\alpha\phi} = \tanh \alpha^2 h t\,. \tag{4.7}$$

We thus see that time starts at $t = 0$ where ϕ is infinite and then continues to $t \to \infty$ where ϕ approaches 0. For the metric we get:

$$e^a = e^{\frac{g}{D-1}t}(\sinh 2\alpha^2 ht)^{\frac{1}{2\alpha^2(D-1)}}\,. \tag{4.8}$$

Thus for $t \to \infty$ we recover the behavior $a \simeq \frac{g+h}{D-1}t \equiv H_+t$, that is the late time de Sitter expansion. However for $t \to 0$ in the past, the metric becomes singular as:

$$e^a \sim t^\gamma\,, \qquad \gamma = \frac{1}{2\alpha^2(D-1)}\,. \tag{4.9}$$

This is a Big Bang dominated by matter with equation of state given by $w = -1 + \frac{2}{D\gamma} = -1 + 4\alpha^2\frac{D-1}{D}$. We thus have $-1 < w \leq 1$, but the precise value depends on the model.

The dS version of the flow of [7] thus represents a cosmology which starts with a Big Bang driven by a specific though model dependent form of matter, and then evolves towards a de Sitter phase. A dual theory which confines in the IR therefore seems to be unsuitable to describe primordial inflation.

A possibility which is in a certain sense opposite to the above one is when we have a potential with an exponential runaway behavior towards zero. This is a well-studied class of potentials in the context of quintessence cosmological models (see [17] for a string theory point of view on quintessence). In our framework, the simplest such potential derives from a prepotential which consists of a single exponential. There are then no extrema and in the RG flow interpretation there are no fixed points. If however we take two exponential terms, W can have a maximum and the RG flow has then a fixed point in the IR but no fixed point in the UV.

A generic exponential prepotential which has a maximum and a runaway behavior is:

$$W = \mu e^{\alpha\phi} - \nu e^{\beta\phi}\,, \qquad \alpha < \beta\,, \tag{4.10}$$

where all the constants are positive. Depending on the parameters, the potential V derived from (2.7) has one of the following three behaviors: i) a positive maximum followed by a negative minimum and then blows up to $+\infty$, ii) a positive maximum and then blows up to $-\infty$ or iii) a negative minimum followed by a positive maximum and then blows up to $-\infty$. Since we are interested in the flows where the potential is positive along all the way, we have to discard the third possibility, which occurs when $2\alpha^2 > \frac{D}{D-1}$. As for the first two options, which differ by analoguous inequalities for β, they are both acceptable as long as we consider a flow going from the maximum to the runaway direction at $\phi \to -\infty$.

Although it is possible to solve exactly these flows (for instance choosing a gauge $b \propto \phi$), the salient features can be extracted from the asymptotic regimes. At early times, such a flow will display an inflationary behavior with de Sitter expansion. The maximum being simple, the flow near it will be similar to the analytic flows discussed at the beginning of the section. At late times, the first term in (4.10) will dominate and the evolution will be quintessence-like like in (4.9), but now for $t \to \infty$. Depending on the exponent α, the spacetime will have a de Sitter-like causal structure in the future if $\gamma > 1$ (that is $w < -1 + \frac{2}{D}$), or a Minkowski-like causal structure in the future if $\gamma \leq 1$, as is discussed in [17]. The problem however arises of whether it is possible to define a dual theory. Indeed if we fall short of a future dS phase, we are unable to relate the dual theory to the future conformal boundary. For the cases with $\gamma \leq 1$, it seems hopeless to try to think in these terms. For the $\gamma > 1$ case, the presence of a future spacelike boundary is encouraging, but it is clear that we cannot use there the same (conformal) tools as for a dS boundary. It would be interesting to understand what kind of UV theory is dual to a quintessence cosmology, if a dual theory can be defined at all. Note that in [18] a correspondence for such cases was proposed (irrespective of the value of γ) along the lines of [19]. The correspondence involves going to a "dual" frame by making an appropriate rescaling of the metric with a function of the scalar. The metric is then of dS form, however the choice of such a dual frame is rather ad hoc in this set up.

The interest of exponential potentials is that they are likely to arise in the low energy effective actions derived from some non supersymmetric versions of string theory. We turn to this analysis in the following section.

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5. Potentials from non supersymmetric strings

The $D+1$ dimensional low energy effective action of a string theory reads in the string frame:

$$S = m_s^{D-1} \int d^{D+1}x \sqrt{-\hat{g}} e^{-2\varphi} \left(\hat{R} + 4 \hat{\partial}\varphi^2 - \mathcal{V}(\varphi) \right) . \tag{5.1}$$

We have restricted the fields present to be only the metric and the ($D+1$ dimensional) dilaton. The action above is derived, for $\mathcal{V}$ constant, from the requirement of quantum conformal invariance of the string world-sheet theory. In a non supersymmetric set up we expect however to have non vanishing dilaton tadpoles at loop orders. Thus a generic potential for the dilaton is:

$$\mathcal{V}(\varphi) = \sum_n c_n e^{n\varphi} = c_0 + c_1 e^{\varphi} + c_2 e^{2\varphi} + \dots . \tag{5.2}$$

The c_0 term is related to a tree level cosmological constant, which arises if the string is in non critical dimensions. Note that even if we fix the dimension D, the sign of c_0 is not fixed. Indeed, allowing for compact directions, the general formula for the superstring is (see for instance [20]):

$$c_0 = \frac{2}{3} \left(D + 1 + \frac{2}{3} c_{\text{int}} - 10 \right) , \tag{5.3}$$

where c_{int} is the WS central charge of the "compact directions". Thus starting with an original non critical superstring theory in dimension greater than 10 and then compactifying down to, say, 4 dimensions, leaves us with a potential which is positive as $\varphi \to -\infty$. This has been used in a related context in [21].

The c_1 term is present if the theory is unoriented and/or has an open string sector, and it has a NSNS tadpole at the disk/crosscap order. The c_2 term originates from a non vanishing torus partition function, that is a one loop cosmological constant, which is likely to be present for a non spacetime supersymmetric theory. Moreover, as noted in [21], a possible further contribution to this order is provided by turning on RR fluxes on the compact space, so that there is even a little room for tuning the parameters.

The above considerations clearly need spacetime supersymmetry to be broken at the string level, however this is perfectly consistent when studying de Sitter backgrounds, which are incompatible with supersymmetry. We only need to worry that there are no tachyons in the theory, at least at tree level. This can be achieved, as for example in the construction of [21].

In order to compare with the general discussion of the previous section, we go to the Einstein frame by the Weyl rescaling of the metric:

$$\hat{g}_{\mu\nu} = e^{\frac{4}{D-1}\varphi} g_{\mu\nu} . \tag{5.4}$$

The action becomes:

$$S = m_p^{D-1} \int d^{D+1}x \sqrt{-g} \left(R - \frac{4}{D-1} \partial\varphi^2 - e^{\frac{4}{D-1}\varphi} \mathcal{V}(\varphi) \right) , \tag{5.5}$$

where we neglect boundary terms. In order to put the action in the form (2.1), we rescale the dilaton by:

$$\varphi = \frac{1}{2}\sqrt{\frac{D-1}{2}}\phi\,, \tag{5.6}$$

and the potential is:

$$V(\phi) \equiv e^{\frac{4}{D-1}\varphi}\mathcal{V}(\varphi) = \sum_n c_n e^{\left(\frac{4}{D-1}+n\right)\frac{1}{2}\sqrt{\frac{D-1}{2}}\phi}\,. \tag{5.7}$$

Before analyzing the conditions for such a potential to be generated by a prepotential, we can already ask which terms can be the leading term. Indeed, a quintessence-like behaviour is possible provided the coefficient in the exponential is not too large. It turns out that the tree level term can always produce quintessence, while the tadpole term (if it is leading) does so only for $D < 9$. This can be seen as follows. It is possible to recast a positive potential $V \sim e^{2\alpha\phi}$ into a prepotential form with $W \sim e^{\alpha\phi}$ if:

$$2\alpha^2 < \frac{D}{D-1}\,. \tag{5.8}$$

The term $n = 0$ of (5.7) always satisfies the bound (which is defined only for $D > 1$). Moreover, from (4.9) we see that it always leeds to $\gamma = 1$, that is the limiting case for which the expansion does not produce a horizon. For the term with $n = 1$, the bound (5.8) is satisfied only for $D < 9$ and the associated γ is smaller than 1, that is the expansion is softer. For instance for $D = 3$ it corresponds to an equation of state with $w = \frac{1}{2}$. Actually $w_{n=1}$ is always positive. It can never be relevant in the future, where it is dominated at least by $w = 0$ matter. This means that if the dilaton potential (5.7) is to be relevant to the late stages of the evolution, it has to include a tree level term.

Let us thus write a prepotential that should generate the first three terms of (5.7), related to those explicitly written in (5.2). Take a prepotential of the form (4.10):

$$W = Ae^{\frac{1}{\sqrt{2(D-1)}}\phi} - Be^{\frac{D+1}{2}\frac{1}{\sqrt{2(D-1)}}\phi}\,. \tag{5.9}$$

The potential is then computed through (2.7):

$$V = A^2 e^{\sqrt{\frac{2}{D-1}}\phi} - ABe^{\frac{D+3}{2\sqrt{2(D-1)}}\phi} - B^2\frac{D-1}{4}e^{\frac{D+1}{\sqrt{2(D-1)}}\phi}\,. \tag{5.10}$$

These can be seen to be the first three terms of (5.7). The condition for a positive V to derive from a prepotential with a maximum is thus that:

$$c_0 > 0\,, \qquad c_1 < 0\,, \qquad c_2 < 0 \qquad \text{and} \qquad c_1^2 = \frac{D-1}{4}c_0|c_2|\,. \tag{5.11}$$

Although the inequalities above can be satisfied in a suitable model, it is difficult to think of a symmetry of the WS theory that would enforce the equality between the constants.[3]

[3] One could still have a little hope of finding a c_2 satisfying the equality in (5.11) by tuning the RR fluxes as in [21], when this is possible.

Of course releasing the constraint one still has a potential with a maximum and a runaway behaviour, but it becomes unaccessible to our tools of analysis (the solution W to (2.7) is not analytic).

Slightly more general potentials were considered in [21], where the aim was actually to find not only a positive maximum but also a positive (or zero) minimum of the potential. This is possible for $c_0 > 0$, $c_1 < 0$ and $c_2 > 0$, this time satisfying a further condition resembling to a loosened version of the equality in (5.11). Such a potential could allow in principle flows similar to the one described at the beginning of the previous section, however it is not possible to describe them analytically.

6. Discussion

We have tried in this paper to discuss issues which arise when one takes seriously the proposal of [6] that the cosmological evolution be interpreted as RG flow in a dual euclidean field theory associated with the future boundary. We found particularly useful to borrow from the tools of the AdS/CFT correspondence the possibility to write first order equations governing the flow, provided the potential derives from a prepotential. In such a preliminary analysis, our perspective has been to review different kinds of cosmological flows and their associated potentials. In particular, quantitative investigations on the dual field theory are still lacking (see however [15] for a recent attempt in this direction).

Having in mind to embed these considerations in string theory, the necessary lack of supersymmetry[4] prevents us from singling out specific models in order to extract predictions. In the context of supergravities, non supersymmetric dS vacua appear in the potential when the theories are "compactified" on non-compact manifolds [24, 23, 25, 18]. One here can make predictions, however the relevance of these theories is not yet clear. More generally, if the supersymmetry is broken to $N = 1$ in 4 dimension, it is easy to generate potentials with (local) dS minima (see for instance [26]). We are then back to the problem of having too much choice.

If we thus break supersymmetry at the string scale, for instance by considering a theory in non critical dimension, possibly with orientifolds (in order to have a negative tadpole on the crosscap), we are left with the potentials discussed in the last section. Here we note that if the dilaton indeed takes the runaway direction, and in the presence of a tree level contribution, the leading behaviour at late times will be $e^a \sim t$, that is the limiting case for which there is no horizon. If we were to consider this as a RG flow, we would have the non-trivial problem of even having to justify the correspondence, since there is no such thing as a future boundary. Any other term in the dilaton potential would lead to an even softer behaviour, as noted in [27]. On the other hand we could consider models like the one of [21], and flow from the maximum to the minimum. Besides that it cannot be studied analytically, we have to be aware of the potential problems in such non supersymmetric theories away from the perturbative limit $\phi \to -\infty$. Not to mention the degree of fine

[4]Note that supersymmetric dS vacua appear in theories with ghosts [22, 23]. For these theories the analysis is much more similar to the AdS case.

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tuning (or luck) needed to find a minimum realizing the enormous hierarchy between the observed cosmological constant and the Planck scale.

As a last remark, one could ask how are to be interpreted flows from a positive maximum of the potential to a negative minimum. This situation is puzzling because it clearly admits both dS and AdS vacua. One could thus ask whether there can be a flow mixing both dS and AdS features.[5] Considering only time evolution, such a flow will start with an inflationary phase and then reach the point where the potential becomes negative. At this point however the scalar cannot possibly settle at the minimum, since this would imply $H^2 < 0$ (we assume flat spacelike sections). Moreover also an oscillatory behaviour cannot be combined with $\dot{H} < 0$. Since when $V < 0$ the scalar field starts to violate the dominant energy condition, $p > \rho$, the expansion is slower and slower and should eventually stop when H reaches zero. After that the universe would start contracting. The AdS cosmology with a big crunch is possibly not reached, but the recontracting behaviour also prevents an holographic interpretation.

Acknowledgments

I would like to thank G. Ferretti and L. Houart for discussions and interesting comments. This work is supported by the European Union RTN contract HPRN-CT-2000-00122.

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